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HEAT TRANSFER IN SHALLOW CROSSFLOW FLUIDIZED BED HEAT EXCHANGERS—I. A GENERALIZED THEORY

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Abstract—A generalized analysis to describe the overall process of gas–particle heat transfer in shallow fluidized bed heat exchangers has been developed. The analysis which takes into account internal particle resistance to heat transfer as well as a residence time distribution and a size distribution of particles may be used for design of new exchangers or for evaluating the performance of existing exchangers. Simpler analyses for certain important special cases were derived from the general analysis by application of the appropriate simplifying assumptions.

NOMENCLATURE

- A, particle surface area per unit base area of bed;
- A_n, A_{sn}, particle surface area and bed wall area in element n;
- B, breadth of bed;
- C_g, C_s, specific heats of gas and particles;
- $E(\theta)\,d\theta$, fraction of particles having reduced bed residence time between θ and $\theta + d\theta$;
- $G(p)\,dp$, fraction of particles having size between p and $p + dp$;
- h, gas particle heat-transfer coefficient;
- L, length of bed;
- M_g, M_s, mass flow rates of gas and particles through bed;
- N, number of elements in analysis;
- Q_n, rate of heat transfer in element n;
- r, R, radius within particle and outside radius of particle;
- t_{amb}, ambient temperature;
- $t_{a_n}, t_{a_{n-1}}$, mean particle temperatures after residence times θ_n and θ_{n-1} in elements n and $n-1$ respectively;
- t_{b_n}, mean bed temperature in element n;
- $t_{gi_n}, t_{g_n}, t_{go_n}$, gas temperature entering element n, at some level in element n and leaving element n;
- $t_n(r), t_{n-1}(\lambda)$, temperature distributions within particles after residence times θ_n and θ_{n-1} in elements n and $n-1$ respectively;
- t_{o_n}, particle surface temperature in element n;
- t_{si}, t_s, t_{so}, particle temperature entering bed, at some position along bed and leaving bed;
- t_{som}, mean particle temperature leaving bed;
- $t_{so\theta}$, particle temperature leaving bed with residence time θ;
- $t_{so\theta p}$, temperature of particles of size p leaving bed with residence time θ;
- t_{soev}, mean particle temperature leaving bed when operating under thermal equilibrium conditions;
- u, superficial gas velocity in bed;
- u_n, superficial gas velocity in element n;
- U, overall heat-transfer coefficient for heat losses;
- W_n, W_T, mass of particles in element n and in the bed as a whole;
- Z, Z_T, depth in bed and total depth of bed.

Greek symbols

- α, thermal diffusivity of particles;
- θ, reduced particle bed residence time;
- θ_n, θ_T, residence time of particles in element n and in bed as a whole;
- λ, radius within particle, referred to element $n-1$;
- ρ, gas density.

1. INTRODUCTION

THE METHOD of cooling or heating particulate solids in a shallow rectangular section fluidized bed is a standard industrial technique. Its use for instance has been reported for the process heating of granular coke [1] and for cooling granular fertilizers [2, 3], sulphur [4] and powdered milk [5]. Practical aspects of design and operation of such units have been discussed by Wormald and Burnell [6].

In spite of the wide use of the technique, heat-transfer analyses have only been developed for operation under special conditions. Thus an analysis for heat transfer in a single stage device under conditions of thermal equilibrium between particles and gas at the top of the bed, and with plug flow of particles through the bed was developed by Kazakova [3]. This was extended by Gelperin and Ainshtein [7] to include the possible invalidity of the thermal equilibrium assumption, by introducing the gas–particle heat-transfer coefficient into the analysis. Borodulya [8] introduced the concept of a particle mixing coefficient but only defined it for the extreme cases of plug flow and perfect mixing. In their analysis Kunii and Levenspiel [9] suggested that the bed be split up into a number of perfect mixers in series with the exit gas temperature being equal to the particle temperature within each mixer. A more general technique for dealing with a particle residence time distribution was proposed and verified by McGaw [10, 11] but his analysis did not include the possibility of particle internal resistance to heat transfer. In the only attempt to allow for particle internal resistance effects Ryazantsev [1] simply applied a correction factor to the inlet particle temperature in Borodulya's [8] analysis. Analyses based on a bed effective thermal conductivity concept have been made [5, 12] but these suffer from the deficiency that internal and external effects are not separated.

In this paper, a general analysis for heat transfer in a shallow crossflow fluidized bed is presented. The analysis takes into account: (a) particle internal resistance to heat transfer; (b) a residence time distribution of particles through the bed; (c) a distribution of particle sizes; (d) uneven inlet air velocity and temperature distributions. The analysis separates internal and external effects by using the particle thermal conductivity and the particle to gas heat-transfer coefficient. Simpler analyses for the important special cases involving the appropriate simplifying assumptions are derived from the general analysis.

2. BASIS OF ANALYSIS

In the development of the general analysis for heat transfer in a shallow crossflow fluidized bed, a numerical method involving internal particle resistance to heat transfer is first developed for the special case of plug flow of uniform sized particles through the bed.

In this initial analysis the full length of bed is divided into a series of elements of equal length in the longitudinal direction. Numerical heat balances are carried out in each element in turn, starting from the particle inlet end and finishing at the particle outlet end of the bed. The width of each element is the width of the bed and the length of each element is determined from the total length of bed divided by the total number of elements. The particles remain in a given element for a particular length of time, θ_n, which is obtained from the following relationship

$$\theta_n = \frac{W_n}{W_T}\theta_T \tag{1}$$

where W_n is the mass of particles in element n, W_T is the total mass of particles in the bed and θ_T is the particle residence time in the bed.

The basic assumptions inherent in the heat balances are as follows: (i) No lateral mixing of gas in the bed. (ii) Perfect mixing of particles in vertical plane.

The heat balance on each element assumes a knowledge of the internal particle temperature distribution, as well as the weight of particles in the element and the gas flow rate. The heat balance calculation is based on the premise that, as soon as the particles enter the element, the particle surface attains a temperature which remains constant for the time the particles are present in the element. The residence time of the particles within each element depends on the number of elements chosen and the particle flow rate, but will generally be only of the order of a fraction of a second. Unsteady-state heat transfer takes place for the duration of the particle residence time in the element and the heat lost from the particles is transferred to the fluidizing gas. The heat balance is used to calculate the mean particle temperature and internal temperature distribution after the particular element residence time, θ_n, these being used as the inlet conditions for the following element. These heat balance calculations are carried out successively from element to element and the outlet particle temperature is the mean particle temperature leaving the final element.

In these heat balance calculations each particle is assumed to have the same treatment so the basic analysis is in effect carried out for plug flow of particles through the bed. This analysis is then appropriately extended to allow for the possibility of a residence time distribution of particles, as well as a distribution of particle sizes.

In the analysis following, the calculation of surface area available for heat transfer and the internal particle conduction heat-transfer calculations were all based on spherical particles. The analysis could be applied in a similar way to other regular shapes by use of the appropriate equations for those shapes.

The analysis presented refers to the cooling of hot particles by a cold gas. A similar approach could be applied to the analogous case of the heating of cold particles by a hot gas.

3. BASIC ELEMENTAL HEAT BALANCE

Consider an element n which is a typical element somewhere along the bed. The conditions in the element are depicted in Fig. 1. The rate of heat transfer from

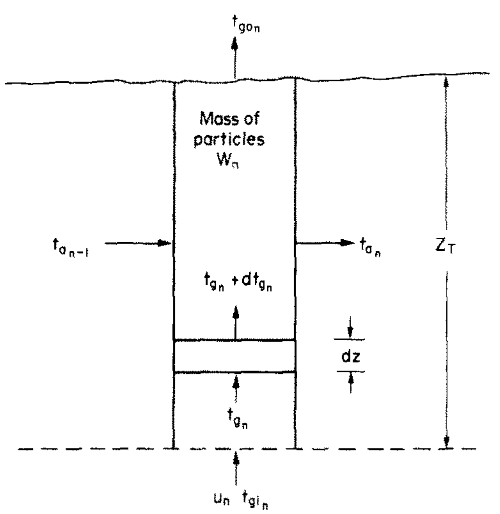

FIG. 1. Elemental heat balance.

the particles in the element Q_n, may be given by the following equation:

$$Q_n = \frac{W_n C_s}{\theta_n}(t_{a_{n-1}} - t_{a_n}) \tag{2}$$

where $t_{a_{n-1}}$ is the mean temperature of particles leaving element $n-1$, t_{a_n} is the mean temperature of particles after contact time θ_n in element n and C_s is the specific heat of the particles. In the application of equation (2), $t_{a_{n-1}}$ will have been calculated from the heat balance in the preceding element, $n-1$. Temperature t_{a_n} will be an unknown and will have to be calculated from a knowledge of the internal temperature distribution. This may be carried out using:

$$t_{a_n} = \frac{3}{R^3}\int_0^R t_n(r).r^2\,\mathrm{d}r \tag{3}$$

where $t_n(r)$ is the temperature within the particles at radius r after contact time θ_n in element n and R is the outside radius of the particles. Since perfect mixing of particles in the vertical plane is assumed, the particles will have a constant surface temperature, t_{o_n}, within the element over contact time θ_n. Thus the relationship between $t_n(r)$ and r may be obtained from the standard unsteady-state heat-conduction heat-transfer relationship for spheres, where the particles possess a specified internal temperature distribution, $t_{n-1}(\lambda)$ at zero contact time. This relationship has been shown [13] to be:

$$t_n(r) = t_{o_n} + \frac{2}{rR}\sum_{m=1}^{m=\infty}\sin\left(\frac{m\pi r}{R}\right)\exp\left(\frac{-\alpha m^2\pi^2\theta_n}{R^2}\right) \times \int_0^R \lambda[t_{n-1}(\lambda)-t_{o_n}]\sin\left(\frac{m\pi\lambda}{R}\right)\mathrm{d}\lambda \tag{4}$$

where λ is the radius within the particle referring to element $n-1$, and α is the thermal diffusivity of the particles.

It is convenient to put this equation in the following form:

$$t_n(r) = A(r)+t_{o_n}[1-B(r)] \tag{5}$$

where

$$A(r) = \frac{2}{rR}\sum_{m=1}^{m=\infty}\sin\left(\frac{m\pi r}{R}\right)\exp\left(\frac{-\alpha m^2\pi^2\theta_n}{R^2}\right) \times \int_0^R \lambda t_{n-1}(\lambda)\sin\frac{m\pi\lambda}{R}\mathrm{d}\lambda \tag{6}$$

and

$$B(r) = \frac{2}{rR}\sum_{m=1}^{m=\infty}\sin\left(\frac{m\pi r}{R}\right)\exp\left(\frac{-\alpha m^2\pi^2\theta_n}{R^2}\right) \times \int_0^R \lambda\sin\frac{m\pi\lambda}{R}\mathrm{d}\lambda. \tag{7}$$

Inserting equation (5) into equation (3) and simplifying gives:

$$t_{a_n} = \eta t_{o_n}+\xi \tag{8}$$

where

$$\eta = \frac{3}{R^3}\int_0^R [1-B(r)]r^2\,\mathrm{d}r \tag{9}$$

and

$$\xi = \frac{3}{R^3}\int_0^R A(r)r^2\,\mathrm{d}r. \tag{10}$$

Substituting for t_{a_n} in equation (2) using equation (8) gives:

$$Q_n = \frac{W_n C_s}{\theta_n}(t_{a_{n-1}}-\eta t_{o_n}-\xi). \tag{11}$$

This is a relationship for the rate of heat transfer from the particles in element n in terms of the mean particle temperature entering from element $n-1$ and the particle surface temperature in element n.

This rate of heat loss by the particles will be equal to the rate of heat transfer to the gas plus the rate of heat loss through the walls of the bed. Thus:

$$Q_n = \frac{\rho u_n LBC_g}{N}(t_{go_n}-t_{gi_n})+UA_{s_n}(t_{b_n}-t_{\mathrm{amb}}) \tag{12}$$

where u_n is the gas velocity in element n, L is the length and B the breadth of bed, N the number of elements in the analysis and ρ the gas density. Temperatures t_{gi_n} and t_{go_n} are the inlet and outlet gas temperatures for the element. U is the overall heat-transfer coefficient for heat loss through the walls which have area A_{s_n} in element n. Temperature t_{b_n} is associated with a mean bed temperature and t_{amb} is the ambient air temperature. Normally the rate of heat loss through the walls is negligible compared to the rate of heat transfer between particles and gas, so that equation (12) reduces to:

$$Q_n = (t_{go_n}-t_{gi_n}).\frac{\rho u_n LBC_g}{N}. \tag{13}$$

Combining equations (11) and (13) gives the following relationship for t_{go_n}

$$t_{go_n} = t_{gi_n} + \frac{M_s C_s N}{\rho u_n LBC_g}(t_{a_{n-1}}-\eta t_{o_n}-\xi). \tag{14}$$

In this relationship the mass flow rate of particles through the bed, M_s, has been substituted for W_n/θ_n as it is more convenient to use.

The particle–gas heat-transfer coefficient h, may be introduced into the analysis by considering a differential heat balance on the gas at some level in the element as depicted in Fig. 1. This balance gives:

$$\frac{\rho u_n LB}{N}.C_g\,\mathrm{d}t_g = hA_n(t_{o_n}-t_{g_n})\frac{\mathrm{d}z}{Z_T}$$

where A_n is the particle surface area in the element available for heat transfer. Since the particles are perfectly mixed in the vertical plane t_{o_n} will be constant and the balance may be integrated directly using the following limits:

$$t_{g_n} = t_{gi_n} \quad \text{at} \quad Z = 0$$

$$t_{g_n} = t_{go_n} \quad \text{at} \quad Z = Z_T.$$

The integration gives:

$$\ln\left[\frac{t_{o_n}-t_{gi_n}}{t_{o_n}-t_{go_n}}\right] = \frac{hA_n N}{\rho u_n LBC_g}.$$

Putting in terms of t_{go_n} gives:

$$t_{go_n} = t_{o_n}-(t_{o_n}-t_{gi})\exp\left[\frac{-NhA_n}{\rho u_n LBC_g}\right]. \tag{15}$$

Eliminating t_{go_n} between equations (14) and (15) gives the following relationship for the unknown t_{o_n}:

$$t_{o_n} = \frac{t_{gi}(1-\Omega)+\kappa(t_{a_{n-1}}-\xi)}{1+\eta\kappa-\Omega} \tag{16}$$

where

$$\Omega = \exp\left[\frac{-NhA_n}{\rho u_n LBC_g}\right] \tag{17}$$

and

$$\kappa = \frac{M_s C_s N}{C_g \rho u_n LB}. \tag{18}$$

In the element to element numerical calculations all the quantities on the R.H.S. of equation (16) are directly calculable, hence t_{o_n} can also be calculated.

Now that t_{o_n} is a known quantity the mean particle temperature in element n can be calculated using equation (8) and the internal particle temperature distribution using equation (4). The outlet gas temperature, if required, can be calculated from either of equations (14) or (15).

4. USE OF ELEMENTAL HEAT BALANCES IN ANALYSIS

In the application of the described heat balance calculation to provide a general analysis for the system as a whole, it is first necessary to specify the particle inlet conditions to the bed. If there is a known internal particle temperature distribution then the described heat balance can be directly applied to element No. 1. However, it will usually be necessary to assume that the particles enter the bed with a constant internal temperature. In this case the heat balance calculations on element No. 1 can be simplified, since the situation in element No. 1 reduces to one of unsteady-state heat conduction in a sphere over a period of time θ_1 with a fixed surface temperature, t_{o_1}, and constant initial internal temperature t_{si}. The relationship for the mean temperature, t_{a_1}, after time θ_1 may thus be given by (13):

$$t_{a_1} = t_{o_1} + (t_{si} - t_{o_1})\frac{6}{\pi^2}\left[\exp\left(\frac{-\pi^2\alpha\theta_1}{R^2}\right) + \frac{1}{4}\exp\left(\frac{-4\pi^2\alpha\theta_1}{R^2}\right) + \ldots\right]$$

which reduces to:

$$t_{a_1} = t_{o_1} + (t_{si} - t_{o_1})C \tag{19}$$

by putting

$$C = \frac{6}{\pi^2}\left[\exp\left(\frac{-\pi^2\alpha\theta_1}{R^2}\right) + \frac{1}{4}\exp\left(\frac{-4\pi^2\alpha\theta_1}{R^2}\right) + \ldots\right].$$

If equation (19) is substituted for t_{a_n} instead of equation (8) in the basic balance, then use of similar procedures to those used in Section 3 gives rise to the following equation from which t_{o_1} can be calculated:

$$t_{o_1} = \frac{\psi t_{si} + (1-\beta)t_{gi}}{1+\psi-\beta} \tag{20}$$

where

$$\psi = \frac{M_s C_s N}{\rho u_n LBC_g}(1-C) \tag{21}$$

and

$$\beta = \exp\left[\frac{-NhA_1}{\rho u_n LBC_g}\right]. \tag{22}$$

The mean outlet particle temperature from element No. 1 may be determined using equation (19) and the particle internal temperature distribution from equation (23):

$$t_1(r) = t_{o_1} + (t_{si} - t_{o_1})\frac{2R}{\pi r}\left[\sin\left(\frac{\pi r}{R}\right)\exp\left(\frac{-\pi^2\alpha\theta_1}{R^2}\right) - \frac{1}{2}\sin\left(\frac{2\pi r}{R}\right)\exp\left(\frac{-4\pi^2\alpha\theta_1}{R^2}\right) + \ldots\right]. \tag{23}$$

These provide the inlet conditions to element No. 2 for which the basic heat balance is used to calculate a mean particle temperature t_{a_2} and internal particle temperature distribution $t_2(r)$. These numerical calculations are carried out successively from element to element until the particles are assumed to leave the bed after element No. N. The mean particle temperature leaving the bed is thus t_{a_N}. The full sequence of calculations is shown in flow diagram form in Fig. 2. This diagram demonstrates the method for determining the mean outlet particle temperature for a situation where particles of a particular size pass through the bed under plug flow conditions.

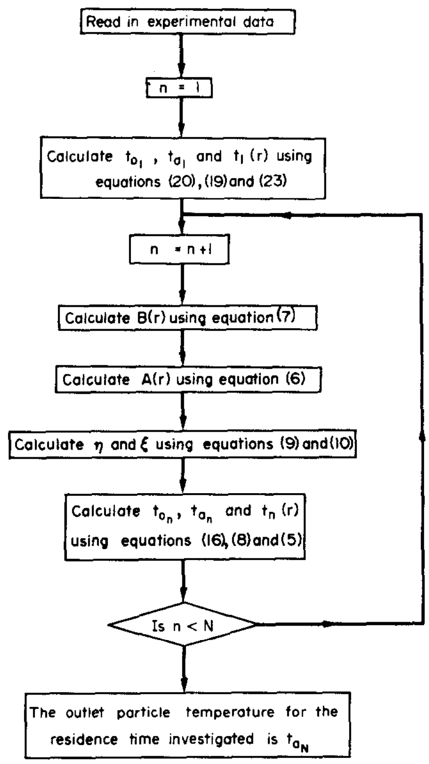

FIG. 2. Sequence for calculating outlet particle temperature.

5. ALLOWANCE FOR PARTICLE RESIDENCE TIME DISTRIBUTION

The analysis described assumes that all particles pass through the bed with the same residence time. In practice this may be approached with a long bed, but in order to generalize the theory it is necessary to allow for the possibility of their being a residence time distribution of particles passing through the bed. This may be done by using an approach similar to that used in chemical reactor theory [14]. Using this approach, the mean temperature of the particles leaving the bed, t_{som}, may be expressed in the following way:

$$t_{som} = \int_0^\infty t_{so\theta} E(\theta)\,d\theta \tag{24}$$

where $t_{so\theta}$ is the mean outlet particle temperature from the bed corresponding to a reduced bed residence time of θ, and $E(\theta)d\theta$ is the fraction consisting of bed residence time between θ and $\theta + d\theta$. This relationship may be used in two different ways depending on the form of the residence time distribution data. If the relationships between $t_{so\theta}$ and θ and $E(\theta)$ and θ are available in equation form, then the integration may be carried out directly. If either or both of the relationships are in numerical form then the integration would have to be carried out numerically as a summation. It should be noted that when there is a residence time distribution of particles, the term M_s should be replaced by M_s/θ in the analysis.

6. ALLOWANCE FOR BOTH PARTICLE RESIDENCE TIME DISTRIBUTION AND PARTICLE SIZE DISTRIBUTION

A particle size distribution may be allowed for in a similar manner to that of the residence time distribution. Thus, if the fraction of particles of a given size is $G(p)dp$, the mean outlet particle temperature for all sizes may be obtained from:

$$t_{som} = \int_0^\infty \int_0^\infty t_{so\theta p} E(\theta) G(p)\,d\theta\,dp \tag{25}$$

where $t_{so\theta p}$ represents the outlet particle temperature for a particular size of particle p, having a reduced bed residence time θ. The particle size distribution will, however, usually be in numerical form so that the integrations would probably be carried out numerically as summations.

7. SPECIAL CASES

The analysis described is a generalized analysis suitable for all situations. There are, however, two industrially important situations for which specific additional assumptions may be made. These additional assumptions simplify the analysis considerably, such that the differential heat balance may be integrated directly over the bed. This results in the overall process of heat transfer being described by a single equation in each case, the use of which is much easier than the general analysis. In each case the inlet gas velocity and temperature is assumed to be constant under the bed.

The first important special case is that where the particles are made from a sufficiently high thermal conductivity material that internal temperature gradients may be neglected, but, the particles are of sufficient size that thermal equilibrium between particles and gas is not achieved in the bed. The second important case is that where the particles are of such small size that internal resistance to heat transfer may be neglected and thermal equilibrium between particles and gas is achieved in the bed.

Special case 1: no internal temperature gradients but no thermal equilibrium

This is a special case where the internal resistance to heat transfer within the particles may be neglected but thermal equilibrium between particles and gas at the top of the bed has not been achieved. Typically, the situation may be one where large particles made of a high thermal conductivity material are processed in a shallow bed.

Reference to equation (4) shows that when the term $(\alpha m^2\pi^2\theta_n/R^2)$ is high, then $t_n(r) \to t_{o_n}$ and internal temperature gradients can be neglected. This may happen when the thermal diffusivity of the particles is high, or when the particle radius is low. For this special case, the heat transfer analysis can be simplified considerably.

Consider element No. n as shown in Fig. 1 as a differential section in the bed. If, as in Section 3, we designate W_n/θ_n to be the mass flow rate of particles through the bed, M_s, and also the temperature difference $(t_{a_{n-1}} - t_{a_n})$ to be dt_s, then the particle rate of heat loss equation (2) simplifies to give:

$$Q_n = M_s C_s\,dt_s. \tag{26}$$

If the length of each element is designated dl, then the number of elements $N = L/dl$. Since the total gas flow rate in the system $M_g = \rho u L B$ then equation (13) for the rate of heat loss by the gas simplifies to:

$$Q_n = \frac{M_g C_g}{L}(t_{go} - t_{gi})\,dl. \tag{27}$$

Combining equations (26) and (27) now gives rise to the following heat balance over the element or differential section:

$$M_s C_s\,dt_s = \frac{M_g C_g}{L}(t_{go} - t_{gi})\,dl. \tag{28}$$

Equation (28) can only be integrated when the relationship between t_{go} and t_s is known. The required relationship may be found by introducing the gas–particle heat-transfer coefficient h, into the analysis using a differential heat balance on the gas at some level in the element, as before. Thus, reference to Fig. 1 shows that:

$$\rho u C_g\,dt_g = hA(t_s - t_g)\frac{dz}{Z_T}$$

where A is the particle surface area per unit base area and t_s is the particle temperature which is constant within the element, there being no internal temperature gradients.

Integrating this over the full depth of bed Z_T gives:

$$\ln\left(\frac{t_s - t_{gi}}{t_s - t_{go}}\right) = \left[\frac{hA}{\rho u C_g}\right].$$

Rearrangement gives:

$$t_{go} - t_{gi} = (t_s - t_{gi})\left[1 - \exp\left(\frac{-hA}{\rho u C_g}\right)\right]. \tag{29}$$

Substituting equation (29) in (27) and combining with equation (26) gives:

$$M_s C_s \, dt_s = \frac{M_g}{L} \cdot C_g (t_s - t_{gi}) \left[1 - \exp\left(\frac{-hA}{\rho u C_g} \right) \right] dl.$$

This can now be integrated directly over the full length of bed using limits

$$1 = L \quad t_s = t_{si}$$

$$1 = 0 \quad t_s = t_{so}$$

to give the following relationship for the outlet particle temperature for this special case:

$$t_{so} = t_{gi} + (t_{si} - t_{gi}) \left\{ \exp \frac{-M_g C_g}{M_s C_s} \times \left[1 - \exp\left(\frac{-hA}{\rho u C_g} \right) \right] \right\}. \quad (30)$$

Equation (30) is the same as that obtained by Gelperin and Ainshtein [7] in their analysis. This equation assumes plug flow of particles through the bed, thus if there is a residence time distribution of particles

$$t_{som} = t_{gi} + (t_{si} - t_{gi}) \int_0^\infty \left[\exp\left\{ \frac{-M_g C_g \theta}{M_s C_s} \times \left[1 - \exp\left(\frac{-hA}{\rho u C_g} \right) \right] \right\} \right] E(\theta) \, d\theta \quad (31)$$

where θ is a dimensionless residence time, defined as residence time divided by mean residence time. If there is a particle size distribution as well as a residence time distribution

$$t_{som} = t_{gi} + (t_{si} - t_{gi}) \int_0^\infty \int_0^\infty \left[\exp\left\{ \frac{-M_g C_g \theta}{M_s C_s} \times \left[1 - \exp\left(\frac{-hA}{\rho u C_g} \right) \right] \right\} \right] E(\theta) G(p) \, d\theta \, dp. \quad (32)$$

Special case 2: thermal equilibrium in bed

For the special case where the product of the heat-transfer coefficient and the surface area per unit base area, hA, is high, then the term $\exp(-hA/\rho u C_g)$ in equation (30) tends to zero. In such a case, the general equation for particle plug flow conditions reduces to:

$$t_{so} = t_{gi} + (t_{si} - t_{gi}) \exp\left(\frac{-M_g C_g}{M_s C_s} \right). \quad (33)$$

This corresponds to the case when thermal equilibrium between the gas leaving the bed and particles at the top of the bed, is achieved. This equation was also obtained by Kazakova [3] who derived it by simply putting $t_s = t_{go}$ in equation (28) and integrating.

For the case where the particle flow through the bed does not approximate a plug flow condition, the residence time distribution may be allowed for in the manner described in Section 5. Thus the mean outlet particle temperature under thermal equilibrium conditions, t_{soev}, may be obtained from:

$$t_{soev} = t_{gi} + (t_{si} - t_{gi}) \int_0^\infty \left[\exp \frac{-M_g C_g \theta}{M_s C_s} \right] E(\theta) \, d\theta. \quad (34)$$

The application of this equation has been verified experimentally [11].

A particle size distribution will not affect heat transfer in this type of situation.

8. DISCUSSION

The general analysis described may be used either (i) as the basis of a design method to determine the size of shallow crossflow fluidized bed necessary to carry out a given heat load, or (ii) to determine the performance of a particular bed operating under given conditions.

If there is a known residence time distribution of particles through the bed, equation (24) must be used in conjunction with the sequence shown in Fig. 2. If there is both a particle size distribution as well as a residence time distribution within the bed then equation (25) must be used in conjunction with the sequence shown in Fig. 2.

In the design case, the particle inlet and outlet temperatures and inlet gas temperature must be specified, together with the particle flow rate, bed operating conditions, gas–particle heat-transfer coefficient and superficial gas velocity. The analysis would then be used to calculate the base area of bed and total gas flow rate necessary to carry out the given load. The method could also be used as a basis for optimizing the size and operating conditions for minimum total cost.

In the determination of the performance of a particular bed operating under given conditions, the gas and particle flow rates must be specified together with the particle and gas inlet temperatures, gas–particle heat-transfer coefficient, and bed conditions. The generalized theory would then be used, possibly in conjunction with either equation (24) or (25), as appropriate, to determine the mean outlet particle temperature from the bed. The method could also be used to optimize the main operating variables, bed depth and gas flow rate, for minimum operating cost.

The two special cases of no particle internal resistance to heat transfer and thermal equilibrium in the bed were shown to be simplified forms of the general analysis. The relationships derived for the particle plug flow condition were the same as those derived by other investigators in different ways [7, 3]. Since standard simple integration techniques were possible in their derivation to produce a single equation in each case, their use in design or performance evaluation is much easier. These solutions may be used, provided that the conditions are known to be such that the appropriate simplifying assumptions are valid. It has been suggested [15] that the Biot number may be used to determine when internal particle thermal resistance effects become important. The critical figure quoted was 0.25. Thus if it is known that $Bi < 0.25$ internal thermal resistance effects may be ingnored and one of equations (30)–(32) may be used for design or performance evaluation as appropriate to the situation. If, however, $Bi > 0.25$ then internal thermal resistance effects cannot be ignored and the general analysis must be used. The second special case assuming thermal equilibrium between particles and gas in the bed is that where the particles are very small. This applies when $\exp(-hA/\rho u C_g) \to 0$ in equations (30)–(32), and in this the simplest possible situation, one of equations (33) and (34) may be used in design or performance evaluation as appropriate. If the inlet air velocity or temperature was uneven over the length of bed it would be necessary to use the general analysis for all situations.

9. CONCLUSIONS

A generalized analysis to describe the process of gas particle heat transfer in shallow crossflow fluidized bed heat exchangers has been developed. Thus the procedures described in Section 4 in conjunction with the proposed methods for incorporating a particle residence time distribution and/or a particle size distribution as described in Sections 5 and 6 may be used as the basis for design or evaluating the performance of existing equipment. When $Bi < 0.25$ particle internal resistance to heat transfer may be neglected and the analysis simplified. A further simplification to the analysis is possible under thermal equilibrium conditions between particles and gas when $\exp(-hA/\rho u C_g) \to 0$.

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TRANSFERT THERMIQUE DANS LES ECHANGEURS DE CHALEUR A LIT FLUIDISE PEU PROFOND—I. THEORIE GENERALE

Résumé—Une méthode généralisée d'analyse a été développée afin de décrire les processus globaux du transfert thermique entre gaz et particules dans les échangeurs de chaleur à lit fluidisé peu profond. L'analyse, qui tient compte de la résistance interne des particules au transfert thermique ainsi que de la distribution des temps de séjour et de celle de la taille des particules, peut être utilisée aussi bien pour la conception de nouveaux échangeurs que pour évaluer les performances des échangeurs existants. Des analyses plus simples, applicables à certains cas particuliers importants, ont été obtenues à partir de l'analyse générale par l'introduction d'hypothèses simplificatrices appropriées.

DER WÄRMEÜBERGANG IN KREUZSTROM-FLIESSBETT-WÄRMEÜBERTRAGERN MIT GERINGER SCHICHTHÖHE—I. EINE VERALLGEMEINERTE THEORIE

Zusammenfassung—Es wird eine verallgemeinerte Theorie zur Beschreibung des Mechanismus des Wärmeübergangs zwischen Gas und Schichtpartikeln in Fließbett-Wärmeübertragern mit geringer Schichthöhe entwickelt. Die Methode, die sowohl den Wärmeleitwiderstand innerhalb der Partikel, wie auch das Verweilzeit- und Größenspektrum der Partikel berücksichtigt, kann zum Entwurf neuer Wärmeübertrager oder zur Berechnung der Leistung bestehender Wärmeübertrager verwendet werden. Aus der verallgemeinerten Theorie werden durch Anwendung geeigneter vereinfachender Annahmen einfachere Berechnungsmethoden für gewisse bedeutende Spezialfälle abgeleitet.

ИССЛЕДОВАНИЕ ТЕПЛООБМЕНА В ТЕПЛООБМЕННИКАХ С ТОНКИМИ ПЕРЕКРЕСТНЫМИ КИПЯЩИМИ СЛОЯМИ

Аннотация — Разработана общая методика описания процесса теплообмена между частицей и газом в теплообменниках с тонкими перекрестными кипящими слоями. Для проектирования новых теплообменников или оценки работы существующих теплообменников может быть использован метод, который учитывает внутреннее сопротивление частиц переносу тепла, а также распределение времени пребывания и размера частиц.

На основе этого метода путем использования соответствующих упрощающих допущений получены более простые варианты для некоторых важных частных случаев.